International Journal of Research in Civil Engineering and Technology

E-ISSN: 2707-8272
P-ISSN: 2707-8264
IJRCET 2022; 3(1): 26-35
Received: 26-11-2021
Accepted: 28-12-2021

Ayinde Muhammed Abdullahi
Department of Mathematics, Faculty of Physical Sciences, Modibbo Adama University, Yola, Nigeria

Ishaq Ajimoti Adam
Department of Physical Sciences, Al-Hikmah University, Ilorin, Nigeria

Oyedepo Taiye
Federal College of Dental Technology and Therapy, Enugu, Nigeria

Alkali Mohammed Adamu
Department of Mathematics, Faculty of Physical Sciences, Modibbo Adama University, Yola, Nigeria

Corresponding Author:
Ayinde Muhammed Abdullahi
Department of Mathematics, Faculty of Physical Sciences, Modibbo Adama University, Yola, Nigeria

Collocation approximation methods for solving higher-order ordinary differential equations

Ayinde Muhammed Abdullahi, Ishaq Ajimoti Adam, Oyedepo Taiye and Alkali Mohammed Adamu

Abstract
In this work, we use the standard collocation method to estimate the solution of higher-order initial value problems (IVPs) in ordinary differential equations (ODEs) by utilizing shifted Chebyshev and shifted Legendre polynomials as basis functions. The ODEs were transformed to integral equations first, then the basis function was substituted to produce a set of linear algebraic equations that were solved using Maple 18. In terms of errors, comparisons were made with the two trial solutions indicated above. To demonstrate the method's performance for varied orders, numerical examples were provided. The shifted Chebyshev polynomial (SCP) basis, on the other hand, outperforms the shifted Legendre polynomial in terms of accuracy, as evidenced by the error tables (SLP).

Keywords: Chebyshev polynomials, integral equations, initial value problem, Legendre polynomials, and standard collocation method

1. Introduction

Ordinary differential equations are used in a wide range of applications, including fluid dynamics, mathematical physics, thermo elasticity, and engineering. Sabo, Kyagya, & Ayinde (2020) [27], Napoli & Costabile (2011) [26], Agbebolu & Omokaro (2010) [5], Akyuz & Sezer (2003) [9], Awoyemi (2003) [12], and Sagir (2012) [28] are only a few of the researchers who have offered numerical solutions to these problems.

A number of problems in theoretical physics, engineering, and other disciplines have recently been discovered to lead to linear and nonlinear integral equations, as well as integro-differential equations and ordinary differential equations. This has drawn the attention of scientists to the solutions to such problems. As a result of this, various methods have been developed in order to find solutions to these problems. These include the hybrid block method developed by Adeniran & Omotoye (2016) [4], single-step hybrid block methods established by Abdelrahim & Omar (2016) [1], the Direct Block method described by Waeleh, Majid, Ismail, & Suleiman (2012) [31], and the Haar Wavelet method developed by Islam, Aziz & Sarler (2010) [19].

Furthermore, Shoukrallah, Nermin, and Ahmed (2021) [29], Mall & Chakraverty (2016) [23], Kayode, Ige, Obarhua, and Omole (2018) [20], Bhrawy & Al-Shomrani (2012) [16], and Akinpelu, Adetunde, & Omiodora (2014) [8] have examined the usage of Lagrange interpolation, Chebyshev, Hermite, Legendre, and As a result, a significant amount of study has been conducted in the field in order to find solutions to these problems. Mbagwu, Cemil, Enyoh & Onwuemeka (2022) [25] also examined the Interpolation and Aitken's method, and Mahmoudi (2005) [27] introduced the Wavelet-Galerkin method. Ahmed, Noorani & Ishak (2008) [6] applied the Homotopy analysis approach, as well as hybrid block methods (Ajileye, Amoo, & Ogwumu, 2018) [6].

$$y' = f(x, y),\ y(a) = y_0, x \in [a,b] \tag{1}$$

Where f is continuous inside the integration interval [a, b], and f satisfies the Lipchitz condition, which ensures that the solution to equation exists and is unique (1). The purpose of using a numerical analyst is to provide an efficient and effective approach for obtaining a numerical solution to problems that are difficult to solve in closed form. Many scholars have used the finite difference approach Burik (2019) [15], the Chebyshev polynomial method

(Taiwo & Olagunju, 2011) [31], Legendre polynomials (Lee & Kung, 1985) [21], and Runge-kutta (Hussain, Ismail & Senu, 2016) [17] to solve ordinary differential equations.
For solving ordinary differential equations, Abualnaja (2015) [2] presented a block technique with a linear multi-step method based on Legendre polynomials. For the numerical solution of fourth-order ordinary differential equations, Areo & Omale (2015) [11] suggested a half-step uniform order symmetric continuous hybrid block technique. For the approximation of fourth-order ordinary differential equations, the interval of integration is taken within a half-step in their work. The schemes and supplementary schemes were created using a collocation and interpolation technique approach.

$$y''' = f\left(x, y, y', y''\right),\ y\left(x_0\right) = \tau_0,\ y'\left(x_0\right) = \tau_1,\ y''\left(x_0\right) = \tau_2 \tag{2}$$

Where x_n is the initial point, y_n is the solution at $x_n,$ f is continuous within the interval of integration.
The above-mentioned researchers' outstanding work inspired us, and eventually led us to solve higher-order linear initial value problems for ordinary differential equations using the standard collocation approximation method, with shifted Chebyshev and shifted Legendre polynomials as our basis functions. We address higher-order ordinary differential equations of the following kind in this work:

$$Ly\left(z\right) = R_m y^{(m)}(z) + R_{m-1} y^{(m-1)}(z) + \cdots + R_2 y^{''}(z) + R_1 y^{'}(z) + R_0 y(z) = f(z) \tag{3}$$

Subject to initial conditions

$$y\left(0\right) = c_0,\ y'\left(0\right) = c_1,\ y''\left(0\right) = c_2, \cdots, y^{(m-1)}(0) = c_{m-1} \tag{4}$$

Where $R^{'s}$ are constants and $f(z)$ is a function of the independent variable.

2. Definitions of Some Relevant Terms

2.1 Collocation method (Adebisi, *et al.*, 2021) [3]

One of the approaches for solving differential equations is the collocation method. It's one of the most used approaches for dealing with ordinary, fractional, and partial differential equations. It entails determining the approximate solution of a given function using an assumed or trial solution. The assumed solution is substituted into the problem, and the resulting equation is collocated (evaluated) at various points within the consideration interval.

2.2 Exact solution (Andrei, 2008) [10]

Exact (closed-form) solutions to mathematical equations play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. A solution is called an exact solution if it can be expressed in aclosed-form such as a polynomial, exponential function, trigonometric function, or the combination of two or more of these elementary functions.

2.3 Approximate solution (Taiwo, Alimi, & Akanmu 2014) [30]

This is the expression obtained after the unknown constants have been found and substituted back into the assumed solution. It is referred to as approximate solution.
In this work, approximate solution used is given a

$$\Phi_N = \sum_{i=0}^{N} c_i Q_i(z) \tag{5}$$

Where $c_i, i = 0,1,2,\cdots,N$ unknown constant to be determined are $Q_i(z)(i \geq 0)$ is the approximating polynomials of any kind, N is the degree of approximant, where in most cases the better approximate solution (i.e. close to the exact solution)

2.4 Chebyshev polynomials (Ayinde & Taiwo, 2017) [14]

$T_i(z)$ stands for the Chebyshev polynomial, which is valid in the interval $a \leq z \leq b$ is defined as

$$T_i(z) = \cos\left[i \cos^{-1}\left(\frac{2z-(a+b)}{b-a}\right)\right], i = 0,1,2,\cdots \tag{6}$$

and the recurrence relation is as follows:

$$T_{i+1}(z) = 2\left(\frac{2z-(a+b)}{b-a}\right)T_i(z) - T_{i-1}(z),\ for\ \mathrm{a \le z \le b} \quad (7)$$

2.5 Conversion of Chebyshev polynomials

In order to change from interval $[a,b] to [0,1]$ the following procedure is carefully followed. Here, we put $a=0$ and $b=1$ in equation (6), we obtain

$$T_i(z) = \cos(\cos^{-1}(2z-1)),[0,1] \quad (8)$$

For the purpose of this work, we need to transform $T_i(z)$ in $[-1,1]$ to $T_i(z)$ in $[0,1]$.

If $z \in [-1,1] \rightarrow v \in [0,1]$ (9)

Let $v = -\gamma + \lambda$, where λ and γ are some parameters. Therefore

$$0 = -\gamma + \lambda \quad (10)$$

$$1 = \gamma + \lambda \quad (11)$$

Solving Eqn. (10) and Eqn. (11) simultaneously, we obtain

$$2\gamma = 1 \Rightarrow \gamma = \frac{1}{2} \quad (12)$$

$$\Rightarrow \gamma = \lambda = \frac{1}{2} \quad (13)$$

$$v = \gamma + \lambda = \frac{z+1}{2} \quad (14)$$

$$\Rightarrow 2v = z+1 \quad (15)$$

By substituting the values of γ and λ, we obtain $z = 2v-1$
Thus,

$$T_i(z) = T_i(\mathrm{2v-1}) \quad (16)$$

Therefore,

$$T_i(v) = \cos(\cos^{-1}(2v-1)),\ for\ 0 \le v \le 1 \quad (17)$$

But v is a dummy variable

$$T_i^*(z) = \cos(\cos^{-1}(2z-1)),\ for\ 0 \le v \le 1 \quad (18)$$

Since

$$T_{i+1}(z) = 2zT_i(\mathrm{z}) - T_{i-1}(z),\ for\ \mathrm{-1 \le z \le 1} \quad (19)$$

Then

$$T_{i+1}^*(z) = 2(2v-1)T_i^*(z) - T_{i+1}^*(z),\ for\ 0 \le \mathrm{z} \le 1 \quad (20)$$

Hence, shifted Chebyshev polynomials is given as

When

$$\left.\begin{array}{l} i=0: T_0^*(z)=1 \\ i=1: T_1^*(z)=2z-1 \\ i=2: T_2^*(z)=8z^2-8z+1 \\ i=3: T_3^*(z)=32z^3-48z^2+18z-1 \\ \vdots \end{array}\right\} \tag{21}$$

Hence, the trial solution is given as

$$y_m=\sum_{i=0}^{M} c_i T_i^*(z) \tag{22}$$

where $c_i, i=0,1,2,\cdots,M$ are unknown constants and $T_i^*(z)$ is the shifted approximating polynomials of first kind. M is the degree of approximating polynomials, where in most cases the better approximate solution (i.e. closer to the exact solution) is produce by larger M .

2.6 Legendre polynomials (Azizul & Shafiqul, 2012) [13, 22]

Legendre polynomial is denoted and defined by the Rodrigue's formula

$$Q_m(z)=\frac{1}{2^m}\frac{\partial^m}{\partial z^m}((z^2-1)^m) \tag{23}$$

From the definition given above, it will be observed that m^{th} derivative must be carried out before a polynomial of degree m is obtained. Thus, the first few sets of Legendre polynomials can be obtained as follows: $Q_0(z)$ will not involve any derivative since $m=0$, hence we obtain $Q_0(z)=1$, Also, for $m=1$, we have the first few Legendre polynomials are given below:

$$\left.\begin{array}{l} Q_m(z)=\frac{1}{2^1}\frac{\partial^1}{\partial z^1}((z^2-1)^1)=z \\ Q_m(z)=\frac{1}{2^2}\frac{\partial^2}{\partial z^2}((z^2-1)^2)=\frac{1}{2}(3z^2-1) \\ \vdots \end{array}\right\} \tag{24}$$

To obtain $Q_3(z)$, it will require differentiating equation (23) three times which might become cumbersome as m increases. With this difficulty that may be encountered with higher differentiation especially as $m>2$ in $Q_m(z)$ of Rodrigue's formula equation (23) above a simple formula for generating the Legendre polynomials is given by its recurrence relation. This is given as equation

$$Q_{m+1}(z)=\frac{2m+1}{m+1}*z*Q_m(z)-\frac{m}{m+1}*Q_{m-1}(z) \tag{25}$$

Since $Q_0(z)$ and $Q_1(z)$ are obtained from Rodrigue's formula equation (23), we can now switch over to equation (25) to generate higher order polynomials as follows:

For $m=1$, we have

$$Q_2(z)=\frac{3}{2}*z*z-\frac{1}{2}*1=\frac{1}{2}(3z^2-1) \tag{26}$$

which is the same as the $Q_2(z)$ earlier obtained using the Rodrigue's formula equation (23)

Thus, for $m = 3,4,\cdots$ we obtain the next few polynomials as follows:

$$Q_3(z) = \frac{1}{2}(5z^3 - 3z) \tag{27}$$

$$Q_3(z) = \frac{1}{2}(35z^4 - 30z^3 + 3) \tag{28}$$

2.7 General shifted Legendre polynomials [22]

The shifted Legendre polynomials are a family of functions that are similar to Legendre polynomials but are defined on the interval $[a,b]$. The well-known Legendre polynomials are defined in the interval $z \in [-1,1]$ and can be calculated using the recurrence formula in equation (2.24) with $Q_0(z) = 1,\ Q_1(z) = z.$

For practical use of Legendre polynomials on the interval of interest $z \in [-1,1]$, it is necessary to shift the defining domain by means of the following substitution

$$z = \frac{2z - a - b}{b - a},\ \ a \le z \le b \tag{29}$$

The shifted Legendre polynomials in z are then obtained as follows

$$Q_0^*(z) = 1 \tag{30}$$

$$Q_1^*(z) = \frac{2z - a - b}{b - a} \tag{31}$$

$$Q_{m+1}(z) = \frac{(2m+1)(2z - a - b)}{(m+1)(b-a)} * Q_m(z) - \frac{m}{m+1} * Q_{m-1}(z) \tag{32}$$

Then, the first few shifted Legendre polynomial is given as

$$\left.\begin{array}{l} i = 0 : Q_0^*(z) = 1 \\ i = 1 : Q_1^*(z) = 2z - 1 \\ i = 2 : Q_2^*(z) = 6z^3 - 6z + 1 \\ i = 3 : Q_3^*(z) = 6z^3 - 30z^2 - 12z + 1 \\ \vdots \end{array}\right\} \tag{33}$$

Hence, the trial solution is given as

$$y_m = \sum_{i=0}^{M} c_i Q_i^*(z) \tag{34}$$

where $c_i, i = 0,1,2,\cdots,M$ are unknown constants to be determined and $Q_0^*(z)$ are the shifted approximating polynomials, M is the degree of approximating polynomials.

3. Problem considered and methodology

In this section, we considered general problems given in eqn. (3), subject to initial conditions given in eqn. (4) above

$$Ly(z) = R_m y^{(m)}(z) + R_{m-1} y^{(m-1)}(z) + \cdots + R_2 y''(z) + R_1 y'(z) + R_0 y(z) = f(z) \tag{35}$$

Subject to initial conditions

$$y(0)=c_0, y'(0)=c_1, y''(0)=c_2, \cdots, y^{(m-1)}(0)=c_{m-1} \tag{36}$$

Where $R^{'s}$ are constants and $f(z)$ is a function of the independent variable.

3.1 Standard collocation method using shifted Chebyshev polynomials

In order to solve equations (35) and (36) using the standard method, we assumed an approximate solution equation (22). Thus, eqn. (22) is differentiated mth-times to get

$$\left.\begin{aligned} y'_m &= \sum_{i=0}^{M} c_i T_i^{*'}(z) \\ y''(z) &= \sum_{i=0}^{M} c_i T_i^{*''}(z) \\ y'''(z) &= \sum_{i=0}^{M} c_i T_i^{*'''}(z) \\ &\vdots \\ y^{(m)}(z) &= \sum_{i=0}^{M} c_i T_i^{*(m)}(z) \end{aligned}\right\} \tag{37}$$

Hence, substituting equations (22) and (37) into equation (35), we obtain

$$\begin{aligned} Ly(z) &= R_m \sum_{i=0}^{M} c_i T_i^{*(m)}(z) + R_{m-1} \sum_{i=0}^{M} c_i T_i^{*(m-1)}(z) + \cdots + R_2 \sum_{i=0}^{M} c_i T_i^{*''}(z) + R_1 \sum_{i=0}^{M} c_i T_i^{*'}(z) \\ &+ R_0 \sum_{i=0}^{M} c_i T_i^{*}(z) \end{aligned} \tag{38}$$

Integrating both sides of equation (38), with respect to u, we obtain

$$\begin{aligned} &R_m \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*(m)}(u) \right) du + R_{m-1} \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*(m-1)}(u) \right) du + \cdots + R_2 \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*''}(u) \right) du \\ &+ R_1 \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*'}(u) \right) du + R_0 \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*}(u) \right) du = \int_0^v \mathrm{f(u)du} \end{aligned} \tag{39}$$

$$\begin{aligned} &\Rightarrow R_m \left[\sum_{i=0}^{M} c_i T_i^{*(m-1)}(v) - \sum_{i=0}^{M} c_i T_i^{*(m-1)}(0) \right] + R_{m-1} \left[\sum_{i=0}^{M} c_i T_i^{*(m-2)}(v) - \sum_{i=0}^{M} c_i T_i^{*(m-2)}(0) \right] + \cdots \\ &+ R_2 \left[\sum_{i=0}^{M} c_i T_i^{*'}(v) - \sum_{i=0}^{M} c_i T_i^{*')}(0) \right] + R_1 \left[\sum_{i=0}^{M} c_i T_i^{*}(v) - \sum_{i=0}^{M} c_i T_i^{*}(0) \right] \\ &+ R_0 \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*}(u) \right) du = \int_0^v \mathrm{f(u)du} \end{aligned} \tag{40}$$

Substituting the conditions given in equation (36) into equation (40), we obtain

$$\begin{aligned} &R_m \left[\sum_{i=0}^{M} c_i T_i^{*(m-1)}(v) - C_{m-1} \right] + R_{m-1} \left[\sum_{i=0}^{M} c_i T_i^{*(m-2)}(v) - C_{m-2} \right] + \cdots + R_2 \left[\sum_{i=0}^{M} c_i T_i^{*'}(v) - C_1 \right] \\ &+ R_1 \left[\sum_{i=0}^{M} c_i T_i^{*}(v) - C_0 \right] \; + R_0 \int_0^v \left(\sum_{i=0}^{M} c_i T_i^{*}(u) \right) du = \int_0^v \mathrm{f(u)du} \end{aligned} \tag{41}$$

Expanding equation (41) further, we obtain

$$R_m\sum_{i=0}^{M}c_iT_i^{*(m-1)}(v)-R_mC_{m-1}+R_{m-1}\sum_{i=0}^{M}c_iT_i^{*(m-2)}(v)-R_{m-1}C_{m-2}+\cdots+R_2\sum_{i=0}^{M}c_iT_i^{*'}(v)-R_2C_1$$
$$+R_1\sum_{i=0}^{M}c_iT_i^{*}(v)-R_1C_0\ \ +R_0\int_0^v\left(\sum_{i=0}^{M}c_iT_i^{*}(u)\right)du=\int_0^v \mathrm{f(u)du} \tag{42}$$

Integrating both sides of equation (42) with respect to v, we obtain

$$R_m\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-1)}(v)\right)dv-R_mC_{m-1}\int_0^z dv+R_{m-1}\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-2)}(v)\right)dv-R_{m-1}C_{m-2}\int_0^z dv+\cdots+R_2\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*'}(v)\right)dv$$
$$-R_2C_1\int_0^z dv+R_1\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*}(v)\right)dv-R_1C_0\int_0^z dv\ \ +R_0\int_0^z\int_0^v\left(\sum_{i=0}^{M}c_iT_i^{*}(u)\right)dudv=\int_0^z\int_0^v \mathrm{f(u)dudv} \tag{43}$$

$$\Rightarrow R_m\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-1)}(v)\right)dv-R_mC_{m-1}z+R_{m-1}\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-2)}(v)\right)dv-R_{m-1}C_{m-2}z+\cdots+R_2\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*'}(v)\right)dv-R_2C_1z$$
$$+R_1\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*}(v)\right)dv-R_1C_0z\ \ +R_0\int_0^z\int_0^v\left(\sum_{i=0}^{M}c_iT_i^{*}(u)\right)dudv=\int_0^z\int_0^v \mathrm{f(u)dudv} \tag{44}$$

Expanding equation (44) further, we obtain

$$R_m\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-1)}(v)\right)dv-R_mC_{m-1}z+R_{m-1}\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-2)}(v)\right)dv-R_{m-1}C_{m-2}z+\cdots+R_2\sum_{i=0}^{M}c_iT_i^{*}(z)$$
$$-R_2C_0-R_2C_1z+R_1\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*}(v)\right)dv-R_1C_0z\ \ +R_0\int_0^z\int_0^v\left(\sum_{i=0}^{M}c_iT_i^{*}(u)\right)dudv=\int_0^z\int_0^v \mathrm{f(u)dudv} \tag{45}$$

This implies

$$\varphi(c_0,z)=\sum_{i=0}^{M}c_iT_i^{*}(z),\ \ \varphi(c_1,z)=\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*}(v)\right)dv,\ \ \varphi(c_2,z)=\int_0^z\left((z-v)\sum_{i=0}^{M}c_iT_i^{*}(v)\right)dv$$

$$\varphi(c_{m-1},z)=\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-1)}(v)\right)dv,\ \varphi(c_{m-2},z)=\int_0^z\left(\sum_{i=0}^{M}c_iT_i^{*(m-2)}(v)\right)dv \tag{46}$$

and

$$F(z)=\int_0^z\int_0^v \mathrm{f(u)dudv} \tag{47}$$

Thus, substituting equation (46) and (47) into equation (45), we obtain

$$R_m\ \varphi(\mathrm{c_{m\text{-}1}},\mathrm{z})-R_mC_{m-1}z+R_{m-1}\varphi(\mathrm{c_{m\text{-}2}},\mathrm{z})-R_{m-1}C_{m-2}z+\cdots+R_2\varphi(\mathrm{c_0},\mathrm{z})$$
$$-R_2C_0-R_2C_1z+R_1\varphi(\mathrm{c_1},\mathrm{z})-R_1C_0z\ \ +R_0\varphi(\mathrm{c_2},\mathrm{z})=F(z) \tag{48}$$

Thus, equation (48) is collocated at point $z=z_k$, to obtain

$$R_m\ \varphi(\mathrm{c_{m\text{-}1}},\mathrm{z_k})-R_mC_{m-1}\mathrm{z_k}+R_{m-1}\varphi(\mathrm{c_{m\text{-}2}},\mathrm{z_k})-R_{m-1}C_{m-2}\mathrm{z_k}+\cdots+R_2\varphi(\mathrm{c_0},\mathrm{z_k})$$
$$-R_2C_0-R_2C_1\mathrm{z_k}+R_1\varphi(\mathrm{c_1},\mathrm{z_k})-R_1C_0\mathrm{z_k}+R_0\varphi(\mathrm{c_2},\mathrm{z_k})=F(\mathrm{z_k}) \tag{49}$$

where

$$\mathrm{z_k}=a+\frac{(b-a)k}{M+1},\ \ \mathrm{k}=1,2,\cdots,\mathrm{M} \tag{50}$$

3.2 Standard collocation method using shifted Legendre polynomials

To discuss this method, we assumed an approximate solution of equation (34). This method is similar to the one we discussed above under subsection (3.1). We have (M+1) algebraic linear system of equation (M+1) unknown constants $c_i(i \geq 0)$. The (M+1)algebraic system of linear are then solved by software Maple 18 to obtain the unknown constants $c_i(i \geq 0)$ which are substituted back in equation (34).

4. Numerical Examples

In this section, we have demostrated the standard collocation approximation method on higher- order ordinary differential equations via shifted Chebyshev and shifted Legendre polynomials as the basis functions and compared it with each other. The examples are solved to illustrate the accuracy, efficiency and time of execution of the method via the software Maple 18.

Example 1. Consider the second order differential equation

$$Ly(z) = 2y''+2y' + y = 0 \quad (51)$$

with initial condition

$$y(0) = 0,\ y'(0) = -1 \quad (52)$$

and exact solution

$$y(z) = 2e^{-z} + z - 2 \quad (53)$$

Example 2. Consider the third order differential equation

$$Ly(z) = y'''+4y' - 5y = 0 \quad (54)$$

with initial condition

$$y(0) = 4,\ y'(0) = -7\ \ y''(0) = 23 \quad (55)$$

and exact solution

$$y(z) = 5 - 2e^{z} + e^{-5z} \quad (56)$$

Example 3. Consider the sixth order differential equation

$$L(y(z)) = y^{(vi)} + 8y^{(v)} + 26y^{(iv)} + 40y''' + 25y'' = 0,\ y^{(v)}(0) = 0 \quad (57)$$

with initial condition

$$y(0) = 4, y'(0) = -1, y''(0) = -3, y'''(0) = 0, y^{(iv)}(0) = 5 \quad (58)$$

and exact solution

$$y(z) = \frac{11}{5} - \frac{21}{5} * z - \frac{2}{5} * e^{-2z}\sin(z) - \frac{11}{5}e^{-2z}\cos(z) - \frac{3}{5}e^{-2z}\sin(z) - \frac{4}{5}e^{-2z}\cos(z) \quad (59)$$

5. Results & Discussion

Table 1: Results and Absolute errors obtained for example 1

z	Exact	SCP Computed Results, M = 6	SLP Computed Results, M = 6	Error$_6$ for SCP.	Error$_6$ for SLP.
0.0	-0.00000	-1.28663×10^{-6}	-1.28662×10^{-6}	1.2866E-06	1.2866E-06
0.2	-0.16254	-0.162538	-0.157980	1.0600E-07	1.0970E-07
0.4	-0.25936	-0.259351	-0.258080	6.0000E-08	5.8900E-08
0.6	-0.30238	-0.302377	-0.309626	4.2000E-08	4.2000E-08
0.8	-0.30134	-0.301342	-0.315619	3.7000E-08	3.7000E-08
1.0	-0.26424	-0.2642411	-0.273639	2.5000E-08	2.5100E-08

Table 1 above, shows numerical solution obtained in term of approximate solution and the errors for the ordinary differential equations solved through shifted Chebyshev and shifted Legendre polynomials basis function. It was observed that the results obtained by the shifted Chebyshev polynomial performed better than shifted Legendre polynomials method in a few iteration and a lower error except in $z = 0.3$ and 0.4

Table 2: Results and Absolute errors obtained for example 2

Z	Exact	SCP Computed Results, M = 6	SLP Computed Results, M = 6	$Error_6$ for SCP.	$Error_6$ for SLP.
0.0	4.00000	3.99312	3.95549	6.8803E-03	3.3771E-02
0.2	2.92507	2.92567	2.88804	5.9498E-04	3.7033E-02
0.4	2.15169	2.15171	2.11417	1.1028E-04	3.7518E-02
0.6	1.40555	1.40552	1.36781	2.5409E-05	3.7653E-02
0.8	0.56723	0.56718	0.52955	5.2795E-05	3.7681E-02
1.0	-0.42983	-0.42992	-0.4675	9.0820E-05	1.4490E-02

Table 2 above, shows numerical solution obtained in term of approximate solution and the errors for the ordinary differential equations solved through shifted Chebyshev and shifted Legendre polynomials basis function. It was observed that the results obtained by the shifted Chebyshev polynomial performed better than shifted Legendre polynomials method in a few iteration and a lower error.

Table 3: Results and Absolute errors obtained for example 3

z	Exact	SCP Computed Results, M = 8	SLP Computed Results, M = 8	$Error_8$ for SCP.	$Error_8$ for SLP.
0.0	-0.0000000000	$3.16069x10^{-5}$	$3.16069x10^{-5}$	9.5017E-08	1.4664E-08
0.2	-0.2596709830	-0.25968	-0.25968	1.7000E-08	3.4000E-08
0.4	-0.6349108848	-0.63491	-0.63491	3.2000E-08	1.7000E-08
0.6	-1.1154622650	-1.11546	-1.11546	3.8400E-08	2.9800E-08
0.8	-1.6869338440	-1.68693	-1.68693	5.2100E-08	4.7800E-08
1.0	-23332466116	-2.33325	-2.33325	7.3500E-08	1.7700E-07

Table 3 above, shows numerical solution obtained in term of approximate solution and the errors for the ordinary differential equations solved through shifted Chebyshev and shifted Legendre polynomials basis function. It was observed that the results obtained by the shifted Chebyshev polynomial performed better than shifted Legendre polynomials method in a few iteration and a lower error.

6. Conclusions

In this work, we have demonstrated collocation approximation method for solving higher-order linear initial value problems (IVPs) of various order using shifted Chebyshev and shifted Legendre polynomials as basis functions. We have also compared our results with each other. The results obtained by shifted Chebyshev polynomial basis performed better over the shifted Legendre polynomial in some examples.

However, we observed that shifted Chebyshev polynomial with the initial approximation obtained from initial conditions yielded a good approximation to the exact solution only in a few iterations.

7. Acknowledgments

I'd like to express my gratitude to the manuscript's handling editor and referee(s) for their informative remarks and ideas.

8. References

1. Abdelrahim R, Omar Z. Direct solution of second-order ordinary differential equation using a single-step hybrid block method of order five. Mathematical and Computational Applications. 2016;21(2):12.
2. Abualnaja K. M. A block procedure with linear multistep methods using Legendre polynomial for solving ordinary differential equations. Journal of Applied Mathematics. 2015;6(1):717-732.
3. Adebisi AF, Uwaheren OA, Ekuk MO, Abolarin OE, Raji MT, Bello KA. Standard collocation method by Legendre basis polynomials function for solving integro-differential equations, The Pacific Journal of Science and Technology. 2021;22(1):37-46.
4. Adeniran AO, Omotoye AE. One step hybrid block method for the numerical solution of general third order ordinary differential equations, International Journal of Mathematical Sciences. 2016;2(5):1-12.
5. Agbebolu GU, Omokaro B. On the solution of singular initial value problems in ordinary differential equations using a new third-order Inverse runge-kutta method. International Journal of Physical Sciences. 2010;5(4):299-307.
6. Ahmed SB, Noorani MSM, Ishak H. Solving systems of ordinary differential equations by homotopy analysis method. Communications in Nonlinear Science and Numerical Simulation. 2008;13(10):2060-2070.
7. Ajileye G, Amoo SA, Ogwumu OD. Two –step hybrid block method for solving first order ordinary differential equations using power series approach, Journal of Advances in Mathematics and Computer Science. 2018;28(1):1-7.
8. Akinpelu FO, Adetunde LA, Omidiora EO. Superiority of Legendre polynomials to Chebyshev polynomial in solving ordinary differential equation, J Appl. Sci. Environ. Mgt. 2005;9(3):121-124.
9. Akyuz A, Sezer M. Chebyshev polynomial solutions of system of higher-order linear differential equations with variable coefficients, Appl. Math. Comput. 2003;144:237-247.

10. Andrei DP, Zaitsev VF. Handbook of exact solutions for differential equations (2nd edition), Chapman & Hall/CRC Press, Boca Raton, 2003.
11. Areo EA, Omole EO. HALF-step symmetric continuous hybrid block method for the numerical solution of fourth order ordinary differential equations. Archives of Applied sciences Research. 2015;7(10):39-49.
12. Awoyemi DO. A p-stable linear multistep method for solving general third-order ordinary differential equations. Int. J. Comput. Math. 2003;80:985-991.
13. Azizur R, Shafiqul I. Legendre piecewise polynomials to approximate the solutions of linear Volterra equations. J. Comput. Math. 2012, 80,
14. Ayinde AM, Taiwo OA. Efficient numerical methods for solving ordinary differential and integro-differential equations. Ph.D. Thesis University of Ilorin, Ilorin, Nigeria, Unpublished. 2017, 14-23.
15. Biruk E. Finite difference method for boundary value problems in ordinary differential Equations, Mathematical Theory and Modeling. 2019;9(10):17-27.
16. Bhrawy AB, Al-Shomrani MM. A shifted Legendre spectral method for Fractional-order multi-point boundary value problems, Advances in Difference Equations. 2012, 8.
17. Hosseini AM. Solving ordinary differential equation using the perturbation term of the tau method over semi-infinite intervals. Far East J Appl. Math. 2000;4(3):295-303.
18. Hussain K, Ismail F., Senu N. Solving directly special fourth-order ordinary differential equations using Runge-Kutta type method Journal of Computational and Applied Mathematics. 2016;306:179-199
19. Islam S, Aziz I, Sarler B., The numerical solution of second-order boundary value problems by collocation method with the Haar wavelets, Mathematical and Computer Modelling. 2010;52:1577–1590.
20. Kayode SJ, Ige OS, Obarhua FO, Omole EO. An order six stormer-cowell-type method for solving directly higher-order ordinary differential equations. Asian Research Journal of Mathematics. 2018;11(3):1-12.
21. Lee H, Kung FC. Shifted Legendre series solution and parameter estimation of linear delayed systems. Int. J.syst. Sci., (1985). 1249-1256.
22. Legendre polynomials" (https://en.wikipedia.org/w/index.php?title=Legendre_
23. polynomials&oldid=1081208123), Retrieved from: file:///C:/Users/user/Downloads/Legendre_polynomials.pdf
24. Mall S, Chakraverty S. Application of Legendre neural network for solving ordinary differential equations, Applied Soft Computing. 2016;43:347-356.
25. Mahmoudi Y. Wavelet Galerkin method for numerical solution of nonlinear integral equations. Applied Mathematics and Computation. 2005;167(2):1119-1129.
26. Mbagwu JPC, Cemil T, Enyoh CE, Onwuemeka JI. Solving second order ordinary differential equations by using Newton’s interpolation and Aitken’s methods, Int. J. Nonlinear Anal, Appl. 2022;13(1):1057-1066.
27. Napoli A, Costabile F. A class of collocation methods for numerical integration of initial value problems (IVPs). Appl. Math. Comput. 2011, 62.
28. Sabo J, Kyagya TY, Ayinde AM. The formation of implicit second order backward difference Adam’s formulae for solving stiff systems of first order initial value problems of ordinary differential equations. Asian Journal of Advanced Research and Reports, 2020;10(4):21-29
29. Sagir AM. An accurate computational of block hybrid method for solving stiff ordinary differential equations. Journal of Mathematics. 2012;4:18-27.
30. Shoukralla ES, Nermin S, Ahmed YS. Computational method for solving weakly singular Fredholm integral equations of the second kind using an advanced barycentric Langrange interpolation formula. 2021, 8(1).
31. Taiwo OA, Alimi AT, Akanmu MA. Numerical solution for linear Fredholm integro-differential difference equations with variable coefficients by collocation methods. Journal of Educational Policy and Entrepreneurial Research. 2014;2(1):175-185.
32. Taiwo OA, Olagunju AS. Chebyshev methods for the numerical solution of fourth order differential equations. Pioneer Journal of Mathematics and Mathematical Sciences. Math. Appl. 2011;147:31-36.
33. Waeleh N, Majid ZA, Ismail F, Suleiman M. Numerical solution of higher-order ordinary differential equations by Direct Block Code. University Putra Malaysia. 2012;8(1):77-81.