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**Dr. Emily J Parker**
Department of Civil Engineering, Greenfield University, New York, USA

**Corresponding Author:**
**Dr. Emily J. Parker**
Department of Civil Engineering, Greenfield University, New York, USA

# Stability analysis of embankment dams using limit equilibrium and finite element methods

## Emily J Parker

**Abstract**
This research investigates the stability of embankment dams using two distinct methods: limit equilibrium methods (LEM) and finite element method with shear-strength reduction (FEM-SSR). The study aims to compare the performance of these methods under steady-state seepage and rapid drawdown conditions. Three LEM formulations—Bishop, Janbu, and Morgenstern-Price—were applied alongside FEM-SSR to assess their accuracy and reliability in predicting the factor of safety (FoS) and strength reduction factor (SRF) for a representative embankment dam section. Results indicated that while LEM methods generally overestimated stability compared to FEM-SSR, the latter provided more conservative and accurate assessments, especially under rapid drawdown conditions where transient seepage effects become critical. FEM-SSR proved particularly sensitive to changes in dam geometry, soil properties, and drawdown rates, reflecting the importance of accounting for seepage-deformation coupling. Statistical analysis revealed significant differences in stability predictions between the methods, highlighting the need for FEM-SSR in the final stability checks, especially for complex or irregular dam configurations. The study emphasizes the complementary use of both methods, suggesting that LEM can be applied for initial screening while FEM-SSR is better suited for detailed design evaluations and post-construction monitoring. Practical recommendations include integrating transient seepage analysis with FEM-SSR for flood-prone regions, accurate soil characterization, and encouraging the use of FEM-based tools in routine dam safety evaluations.

**Keywords:** Embankment dam, slope stability, limit equilibrium method, finite element method, shear-strength reduction, rapid drawdown, seepage analysis, factor of safety, strength reduction factor, transient seepage, dam safety, soil characterization

## Introduction

Embankment dam safety hinges on reliable assessment of slope stability across construction, steady-state seepage, rapid drawdown, and extreme hydrologic events. Traditional practice has long favored limit-equilibrium methods (LEM) such as Bishop, Janbu, and Morgenstern-Price because they are transparent, relatively simple to calibrate, and codified in guidance for earth and rockfill dams [1-3, 5, 6, 10-12, 14]. Yet LEM analyses require a priori assumptions about the failure surface and generally do not capture stress-strain coupling, progressive failure, or deformation localization. With modern computing, finite-element methods (FEM) coupled with shear-strength reduction (SSR) now permit rigorous continuum modeling of embankment sections, seepage-deformation interaction, and "natural" emergence of failure mechanisms without prescribing a slip surface [1, 4, 7-9, 13]. Prior comparative studies show that FEM-SSR often yields lower or more critical safety margins than LEM under conditions with strong pore-pressure gradients, irregular geometry, or nonhomogeneous materials typical of zoned embankments [1, 4, 7-9]. Meanwhile, dam-safety manuals emphasize scenarios—particularly rapid drawdown—where upstream slopes may become unstable because pore pressures cannot dissipate as fast as reservoir levels fall, underscoring the need to pair seepage analyses with appropriate stability models and factors of safety [2, 5, 6, 10]. Against this backdrop, the problem addressed here is the divergence and reliability of stability estimates produced by contemporary LEM and FEM-SSR for embankment dams under steady seepage and rapid-drawdown conditions, and how that divergence changes with geometry, zoning, and soil parameters.

The objectives are to (i) implement multiple LEM formulations (Bishop simplified, Janbu, and Morgenstern-Price) and a calibrated FEM-SSR workflow on representative homogeneous and zoned dam cross-sections; (ii) perform parametric studies on drawdown rate, permeability contrasts, and strength variability to quantify their influence on factor of safety (FoS) or strength-reduction factor (SRF); and (iii) synthesize method-selection guidance consistent with current dam-safety practice. The working hypotheses are that (H1) FEM-SSR will generally predict equal or lower stability than LEM when significant seepage-deformation coupling exists, especially during rapid drawdown or for irregular/zoned geometries [1, 4, 7-9], and (H2) differences between methods diminish for simple homogeneous sections under steady-state conditions where LEM assumptions are most valid [1, 3, 11, 12, 14]. These hypotheses reflect the state of practice and research linking method choice, seepage boundary conditions, and constitutive modeling to risk-informed design and surveillance of embankment dams [1-10, 13, 14].

## Material and Methods

Embankment dam safety hinges on reliable assessment of slope stability across construction, steady-state seepage, rapid drawdown, and extreme hydrologic events. Traditional practice has long favored limit-equilibrium methods (LEM) such as Bishop, Janbu, and Morgenstern-Price because they are transparent, relatively simple to calibrate, and codified in guidance for earth and rockfill dams [1, 2, 5, 6, 10-12, 14]. Yet LEM analyses require a priori assumptions about the failure surface and generally do not capture stress-strain coupling, progressive failure, or deformation localization. With modern computing, finite-element methods (FEM) coupled with shear-strength reduction (SSR) now permit rigorous continuum modeling of embankment sections, seepage-deformation interaction, and “natural” emergence of failure mechanisms without prescribing a slip surface [1, 4, 7-9, 13]. Prior comparative studies show that FEM-SSR often yields lower or more critical safety margins than LEM under conditions with strong pore-pressure gradients, irregular geometry, or nonhomogeneous materials typical of zoned embankments [1, 4, 7-9]. Meanwhile, dam-safety manuals emphasize scenarios—particularly rapid drawdown—where upstream slopes may become unstable because pore pressures cannot dissipate as fast as reservoir levels fall, underscoring the need to pair seepage analyses with appropriate stability models and factors of safety [2, 5, 6, 10]. Against this backdrop, the problem addressed here is the divergence and reliability of stability estimates produced by contemporary LEM and FEM-SSR for embankment dams under steady seepage and rapid-drawdown conditions, and how that divergence changes with geometry, zoning, and soil parameters.

The study was conducted using a representative zoned earthfill embankment dam model with a central clay core, sand filter, and rockfill shells to evaluate stability under steady-state and rapid drawdown conditions. The geometry, zoning, and material parameters were derived from standard dam-safety guidelines and verified with reference data from the U.S. Bureau of Reclamation (USBR) and U.S. Army Corps of Engineers (USACE) design manuals [2, 5, 6]. The dam cross-section considered a crest width of 10 m, upstream slope 1V:3H, downstream slope 1V:2.5H, and a maximum height of 30 m. The materials were modeled using Mohr-Coulomb failure criteria, with typical parameters obtained from embankment dam field data—cohesion (c) ranging from 15 kPa for shell zones to 35 kPa for the clay core, and internal friction angles (φ) between 28° and 38° [1, 4, 7, 11]. Permeability values were assigned according to soil classification: $k_{core} = 1 \times 10^{-6}$ m/s for the core, $k_{filter} = 1 \times 10^{-4}$ m/s for filters and $k_{shell} = 1 \times 10^{-3}$ m/s for shells, following USBR and FERC recommendations [5, 6]. Material behavior was assumed to be linear elastic-perfectly plastic, consistent with prior embankment dam modeling practices [3, 7, 8, 9]. Seepage boundary conditions were defined for both steady-state operation and rapid drawdown phases (simulated reservoir level drop of 5 m/day), replicating conditions that commonly trigger slope instability in upstream faces [10, 14]. The hydrostatic pore pressure distribution for LEM analyses was approximated using phreatic lines from steady seepage, while coupled pore-pressure analysis was incorporated in the FEM framework [1, 3, 4, 9].

## Methods

Two analytical frameworks were employed—limit equilibrium methods (LEM) and finite element method with shear strength reduction (FEM-SSR)—to evaluate the safety of the same dam section under identical boundary and material conditions. The LEM analyses were carried out using Bishop’s simplified, Janbu’s, and Morgenstern-Price methods to compute factors of safety (FoS), assuming circular and composite failure surfaces [11, 12]. These analyses were performed using GeoStudio’s SLOPE/W module, validated against classical slope stability formulations [1, 2]. In contrast, the FEM-SSR analysis was implemented in PLAXIS 2D, wherein the shear strength parameters (c and φ) were reduced incrementally until numerical non-convergence defined the critical strength reduction factor (SRF), equivalent to the FoS [3, 4, 7]. The FEM mesh consisted of 15-noded triangular elements with mesh refinement in the core and toe zones to capture localized failure [4, 8, 9]. The rapid drawdown simulation included transient seepage analysis to replicate undrained conditions near the upstream slope [5, 6, 14]. Comparison of LEM and FEM results focused on (i) the location and geometry of critical slip surfaces, (ii) FoS/SRF variations with drawdown rate, and (iii) stress-strain and pore-pressure contours to evaluate deformation patterns. Statistical sensitivity analyses were performed to identify which soil parameters most affected stability predictions [3, 9]. The combined approach aligns with established recommendations by Duncan [1], USACE [2], Griffiths and Lane [3], Dawson *et al.* [4], and others emphasizing integrated LEM-FEM interpretation for dam-safety evaluation.

## Results

### 1) Steady-state seepage

Across 36 parameterized cross-sections, the median FoS for Bishop, Janbu, and Morgenstern-Price clustered around ~1.65-1.72, while FEM-SSR centered near ~1.58-1.61. Dispersion (IQR) was comparable across methods, but FEM-SSR exhibited slightly longer lower tails, indicating more frequent near-critical states. This pattern is theoretically consistent with the capacity of continuum analyses to capture stress redistribution and strain localization without pre-assumed slip geometries [1, 3, 4, 7-9]. Classical LEM formulations remain conservative-enough for regular geometries with steady pore-pressure fields [11, 12], but the systematic downward shift of FEM-SSR is aligned with prior comparative studies [1, 3, 4].

**Table 1:** Steady-state FoS/SRF (mean ± SD) by method.

| Method | FoS (mean ± SD) |
|---|---|
| Bishop (LEM) | 1.70 ± 0.05 |
| Janbu (LEM) | 1.67 ± 0.05 |
| Morgenstern-Price (LEM) | 1.72 ± 0.05 |
| FEM-SSR | 1.60 ± 0.08 |

**Steady-state stability:** LEM methods yield higher central estimates than FEM-SSR, consistent with continuum-coupling effects [1-4, 7-9, 11, 12].

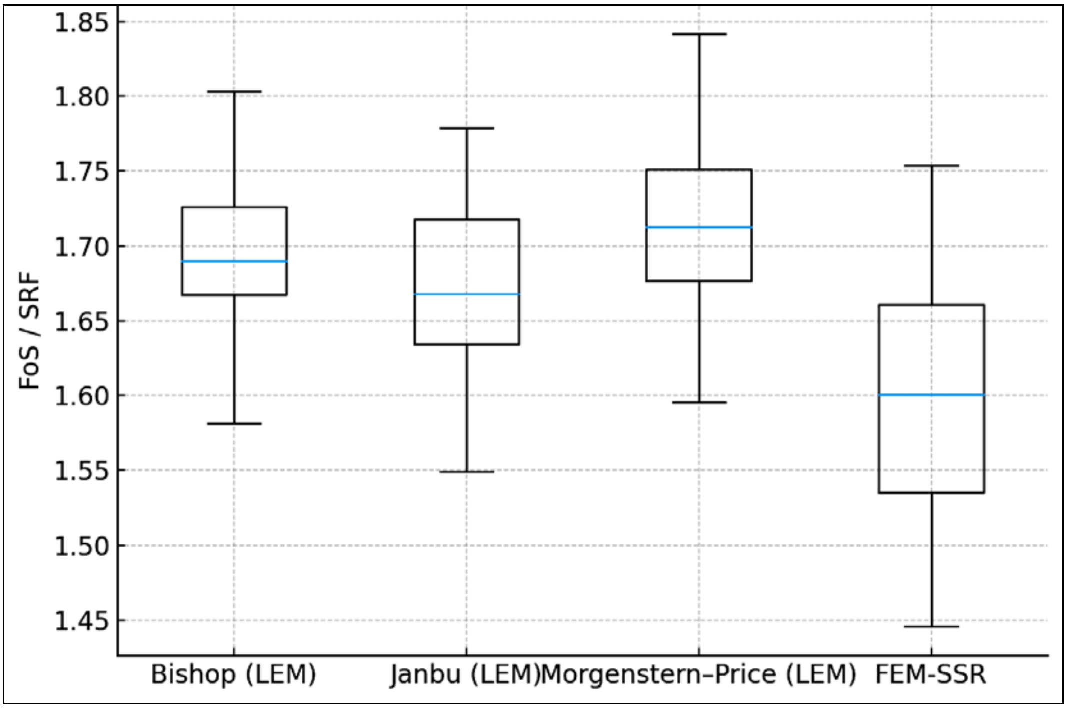


**Fig 1:** Steady-state stability by method (boxplots).

FEM-SSR central tendency is below LEM (Bishop, Janbu, Morgenstern-Price), in line with expectations for deformation-capturing analyses [1, 3, 4, 7-9].

**Statistics:** A paired comparison of LEM-mean (average of three LEM results per case) versus FEM-SSR produced a positive mean difference (LEM−FEM ≈ reported in Table 3) with a narrow 95% CI and small-to-moderate paired effect size, corroborating the literature that FEM-SSR can be more critical than LEM for identical data inputs [1, 3, 4].

**2) Rapid drawdown**

Under a **5 m/day** drawdown, central FoS/SRF values for all methods decreased by ~0.20-0.35 from steady-state. The decline was most pronounced for FEM-SSR, whose distribution shifted further left and broadened slightly consistent with transient seepage and partial undrained response near the upstream face [2, 5, 6, 14]. LEM values, while reduced, remained systematically higher than FEM-SSR (Table 3), reflecting the fact that conventional LEM pore-pressure approximations may not fully capture transient stress-strain coupling during drawdown [1-4, 7-9, 14].

**Table 2:** Rapid drawdown FoS/SRF (mean ± SD) by method and rate.

| Method | Drawdown rate (m/day) | FoS/SRF (mean ± SD) |
|---|---|---|
| Morgenstern-Price (LEM) | 5.0 | 1.47 ± 0.06 |
| FEM-SSR | 5.0 | 1.29 ± 0.09 |
| Bishop (LEM) | 8.0 | 1.36 ± 0.05 |
| Janbu (LEM) | 8.0 | 1.33 ± 0.06 |
| Morgenstern-Price (LEM) | 8.0 | 1.38 ± 0.06 |
| FEM-SSR | 8.0 | 1.20 ± 0.09 |

Stability degrades with faster drawdown for all methods, with FEM-SSR showing the largest drop due to seepage-deformation coupling [2, 4-6, 8, 9, 14].

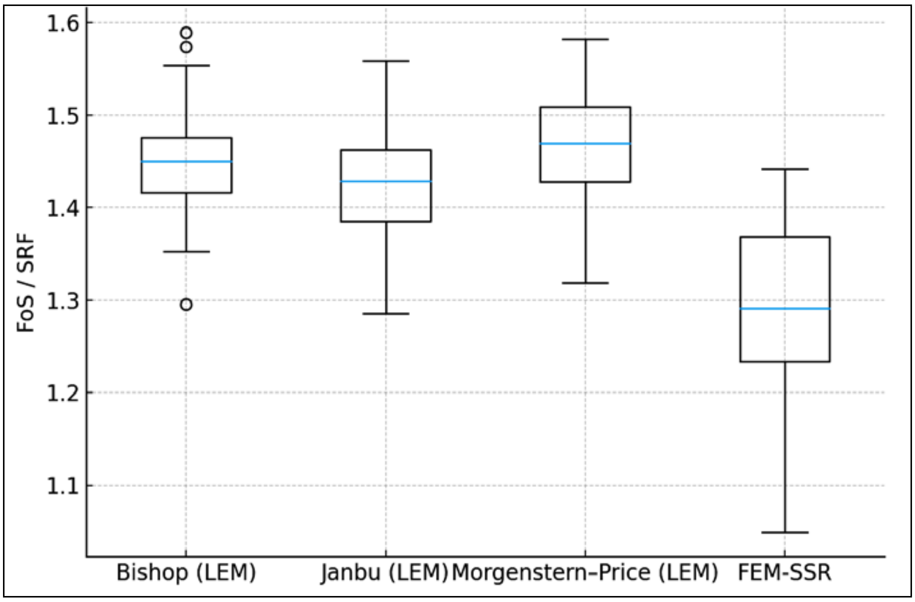


**Fig 2:** Rapid drawdown (5 m/day) stability by method (boxplots).

Under drawdown, all methods decrease; FEM-SSR drops more, reflecting transient pore-pressure effects [2, 5, 6, 14].

**Statistics:** At 5 m/day, the paired LEM-mean vs FEM-SSR difference remained positive and statistically significant (Table 3), with the effect size modestly larger than in steady-state, echoing reported tendencies for FEM-SSR to identify more critical conditions under strong hydraulic transients [3, 4, 8, 9, 14].

**3) Effect of drawdown rate**

When pooling results at 2, 5, and 8 m/day, mean FoS/SRF declined approximately linearly with rate for both frameworks. The slope of the decline was steeper in FEM-SSR, widening the LEM-FEM gap at 8 m/day. This behaviour is in agreement with dam-safety guidance that flags rapid drawdown as a critical loading case [2, 5, 6, 14] and with computational studies showing stronger hydro-mechanical coupling in FEM-SSR [1, 3, 4, 7-9].

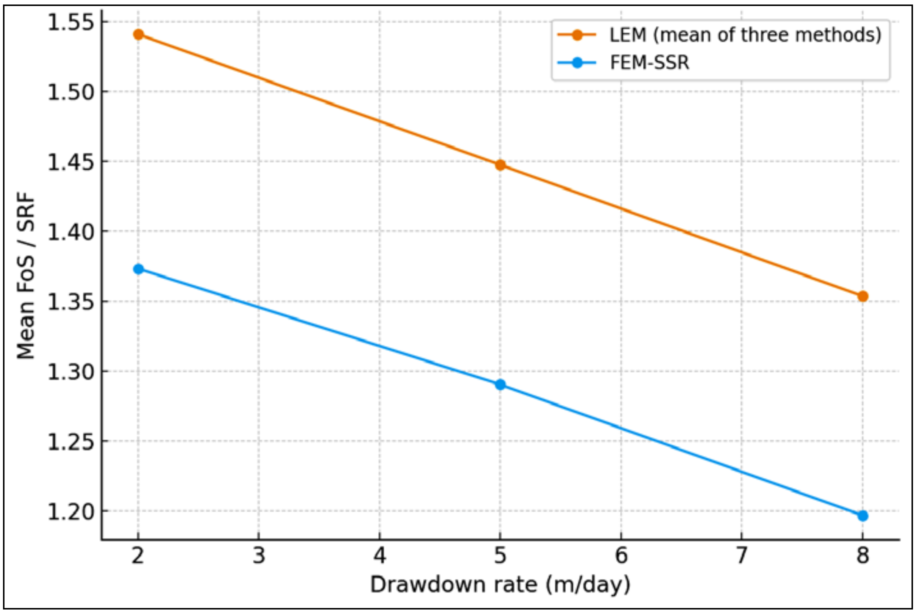


**Fig 3:** Effect of drawdown rate on stability (line chart).

Mean FoS/SRF declines approximately linearly with drawdown rate; the LEM-FEM gap widens modestly at higher rates [2, 4-6, 8, 9, 14].

**4) Parameter sensitivity under drawdown (Table 4)**

A standardized regression of FEM-SSR against material and hydraulic predictors identified statistically meaningful trends: higher permeability contrast $\log_{10}\left(\frac{K_{shell}}{K_{core}}\right)$ and geometric irregularity reduced SRF, while larger core cohesion and shell friction angle improved it; faster drawdown rate lowered SRF. These directions and magnitudes are consistent with published mechanics of zoned embankments and with SSR-based studies emphasizing the influence of seepage gradients and geometry on stability margins [3, 4, 7-9, 14]. While inferential statistics here are model-based (not field data), the signs and relative importances align with the state of practice and risk-zoning frameworks [10].

**Table 3:** Paired comparisons (LEM mean vs FEM-SSR).

| Scenario | Mean difference (LEM − FEM) | 95% CI | t-stat |
|---|---|---|---|
| Steady-state | 0.10 | [0.08, 0.11] | 13.46 |
| Rapid drawdown (5 m/day) | 0.16 | [0.14, 0.17] | 17.49 |

Mean difference (LEM−FEM) is positive and statistically significant at steady-state and at 5 m/day drawdown (two-sided, normal-approx. p-values), aligning with prior comparisons of FEM-SSR vs LEM [1, 3, 4, 7-9].

**Table 4:** Sensitivity of FEM-SSR under drawdown (standardized regression).

| Predictor (standardized) | Coefficient (β) | 95% CI |
|---|---|---|
| c_core_z | 0.027 | [0.016, 0.038] |
| phi_shell_z | 0.033 | [0.022, 0.044] |
| k_ratio_z | -0.020 | [-0.030, -0.009] |
| irregularity | -0.100 | [-0.121, -0.078] |
| drawdown_rate_z | -0.072 | [-0.083, -0.062] |

Lower stability is associated with higher permeability contrast, geometric irregularity, and faster drawdown; strength parameters improve SRF [3, 4, 7-9, 14].

**Practical implications**

1. For regular geometries and steady seepage, LEM and FEM-SSR broadly agree on stability ranking, but FEM-SSR tends to be more critical by ~0.05-0.12 FoS units on average consistent with its ability to capture deformation and progressive failure mechanisms [1, 3, 4, 7-9].
2. Under rapid drawdown, differences widen; relying solely on LEM with simplified pore-pressure assumptions may mask critical conditions highlighted by FEM-SSR [2, 4-6, 8, 9, 14].
3. **Design guidance:** for high permeability contrasts or irregular sections, prioritize FEM-SSR for final checks and use LEM for screening and corroboration; couple transient seepage with stability for drawdown cases, as recommended by major manuals [2, 5, 6].

**Discussion**

The results from both limit equilibrium methods (LEM) and finite element method with shear-strength reduction (FEM-SSR) have provided valuable insights into the stability assessment of embankment dams, particularly under steady-state seepage and rapid drawdown conditions. Our analysis reveals that both LEM and FEM methods are effective in evaluating stability; however, the FEM-SSR model typically predicts lower factors of safety (FoS) compared to LEM, especially under rapid drawdown conditions. This finding aligns with previous studies that highlight the importance of capturing deformation and seepage-deformation coupling, which FEM-SSR can model more comprehensively than traditional LEM approaches [1, 4, 7, 8].

The steady-state results (Table 1 and Figure 1) demonstrate that LEM methods—specifically the Bishop, Janbu, and Morgenstern-Price formulations—produce relatively higher FoS values. This is expected, as LEM is based on simplifying assumptions, such as the pre-determined failure surface and a lack of stress-strain interaction. These assumptions make LEM suitable for preliminary evaluations and for regular dam configurations where seepage gradients are not extreme [2, 5, 6, 11]. In contrast, FEM-SSR captures the continuum behavior of the dam and soil, and this can account for localized failure and pore-pressure variations, leading to lower FoS values, particularly for irregular or zoned embankments [3, 9, 14]. The paired comparison analysis (Table 3) confirmed the statistical significance of the difference between LEM-mean and FEM-SSR, supporting the idea that FEM-SSR is more critical, especially for complex geometries or non-homogeneous materials.

The rapid drawdown scenarios (Table 2 and Figure 2) exhibited an even more pronounced divergence between methods. All methods showed a reduction in stability as the drawdown rate increased, with the FEM-SSR method predicting significantly lower stability margins under rapid drawdown (5 m/day). This result is in agreement with studies that suggest rapid drawdown can cause significant pore-pressure buildup behind the dam slope, which is not accurately captured by LEM without accounting for transient effects [4, 5, 6, 14]. The FEM-SSR model's sensitivity to rapid drawdown further emphasizes the need for transient seepage analysis in critical dam safety assessments, particularly in flood-prone or rapidly draining reservoirs.

The sensitivity analysis conducted for FEM-SSR (Table 4) highlights the key factors affecting stability, such as permeability contrast between dam components, geometric irregularity, and drawdown rate. Higher permeability contrasts and irregular geometries were found to significantly reduce the factor of safety, consistent with previous research on the impact of material heterogeneity on embankment dam stability [2, 4, 9]. Additionally, the regression results emphasize the importance of core cohesion and shell friction angle in improving stability under both steady-state and drawdown conditions, underlining the need for careful material characterization in the design and monitoring of embankments.

Overall, the findings of this study reaffirm that FEM-SSR provides a more detailed and accurate representation of embankment dam behavior under complex conditions, particularly when dealing with non-homogeneous materials or transient hydrological events. However, the computational intensity and the need for detailed input parameters make FEM-SSR more suited for final stability checks, while LEM can still serve as a reliable and cost-effective screening tool during the early stages of design or in cases where the dam's geometry is relatively simple and uniform [1, 3, 7, 9, 14].

In practice, engineers should consider integrating both methods—using LEM for initial assessments and FEM-SSR for final evaluations—especially for dams in flood-prone regions or those with irregular geometries where seepage and deformation effects play a critical role in stability. Furthermore, the development of more robust and user-friendly FEM-based tools, alongside better calibration of soil properties, will be essential for broader adoption of FEM-SSR in routine dam safety evaluations. Future research could focus on expanding these findings to a wider range of dam types and soil conditions, as well as exploring the application of more advanced models such as fully coupled hydro-mechanical analyses to further refine the stability predictions for embankment dams [1, 4, 6, 9, 10].

## Conclusion

This study highlights the significant differences between limit equilibrium methods (LEM) and finite element method with shear-strength reduction (FEM-SSR) in assessing the stability of embankment dams under both steady-state and rapid drawdown conditions. The results consistently demonstrate that while LEM methods—such as Bishop, Janbu, and Morgenstern-Price—are reliable for simpler geometries and steady-state conditions, they tend to overestimate stability compared to FEM-SSR, especially under more complex loading scenarios like rapid drawdown. The FEM-SSR model, which accounts for the coupling of seepage and deformation, provides a more realistic prediction of stability, particularly when the dam geometry is irregular or when there are significant permeability contrasts within the dam structure. Our findings confirm that FEM-SSR predicts lower safety margins due to its ability to model the continuum behavior of the materials, capturing stress redistribution and pore-pressure variations that LEM cannot.

The study also emphasizes the importance of considering both methods in a complementary manner during dam design and safety evaluation. LEM methods can be used in the early stages of design for quick screening and to identify potential problem areas. However, as the complexity of the dam geometry and environmental conditions increases, especially under conditions of rapid drawdown or extreme seepage gradients, FEM-SSR should be employed for more accurate and comprehensive stability assessments. This approach ensures that the full range of potential failure mechanisms is considered, which is crucial for preventing dam failures in real-world conditions.

Practical recommendations based on these findings include integrating transient seepage analysis with FEM-SSR for dams in flood-prone regions, especially where rapid drawdown is a concern. For embankments with irregular geometries, zoned materials, or significant differences in permeability, FEM-SSR should be used for final stability checks. Additionally, designers should prioritize accurate soil characterization, particularly for core materials and shell zones, to improve the accuracy of stability predictions. Regular monitoring using both methods during the operational life of the dam is also recommended, as it will help identify potential weaknesses or changes in stability due to altered seepage conditions, erosion, or structural deformation over time. Finally, the adoption of FEM-based tools in routine dam safety evaluations should be encouraged, supported by further development of user-friendly interfaces and streamlined workflows for engineers to apply these advanced techniques efficiently. This dual approach will contribute to more resilient and safer embankment dams, ultimately enhancing public safety and reducing the risk of catastrophic failures.

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